



Cambridge International Examinations

Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS

9709/52

Paper 5 Probability & Statistics 1

February/March 2023

MARK SCHEME

Maximum Mark : 50



Cambridge Assessment
International Education

[Turn over

Cambridge Assessment International Education – Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
.
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
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- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Guidance For Examiners

Preparation For Marking

1. Please familiarise yourself with the RMA3 guide entitled *Examiners' Instructions on screen* (alternatively called *Instructions for examiners (marking on-screen)*). This may be found among *Assistant Examiner RM Assessor3 online marking resources* through the training site: <https://training.cambridgeassessment.org.uk/login/index.php>
2. If this is the first time you have marked using RMA3 please familiarise yourself with the *RM Assessor3 How To Guides* on the training site. If you have marked using RMA3 before you may wish to refamiliarise yourself these guides.
3. Work through the question paper and **provisional** mark scheme. There will be further amendments and clarifications to this mark scheme at the Standardisation Meeting.
4. Try to be clear in your own mind of what is required for each mark, and in particular for M (method) marks.
5. Browse scripts in RMA3 – see section C3 of the *Instructions for examiners (marking on-screen)*. These should be available approximately 9 days after the examination is taken, however it is worth checking after about 7 days. Please look at about 10 scripts and refer any queries/issues to your Team Leader for consideration at the standardisation team meeting.

Standardisation

1. A **FINAL** mark scheme will be issued at/after the Standardisation meeting. You **must** use this version of the mark scheme for your marking.
2. Mark the 10 practice scripts and check them, ensuring the Comments Box is open – see Section C4 of the *Instructions for examiners (marking on-screen)*.
3. Mark the 10 standardisation scripts and submit them to your Team Leader- see Section C5 of the *Instructions for examiners (marking on-screen)*.
4. Your Team Leader will feed back to you and approve you to proceed with marking or ask you to mark a further 10 scripts.

Marking Guidance

1. If you are in doubt about applying the mark scheme consult your Team Leader, preferably using the RMA3 messaging system.
2. There must be evidence that all of a candidate's work has been marked. Any blank pages must be checked for responses by scrolling down and then each annotated with a BP or SEEN. Similarly, if you receive a script with Additional Objects please annotate each page with SEEN to show that you have checked these pages.
3. Responses that are not awarded either 0 or full marks should be appropriately annotated.
4. Although the scheme will allow for the majority of likely methods, full marks are to be given for a correct result from any correct method, with equivalent sub-marks for equivalent stages. The abbreviation AEF in the mark scheme indicates that any equivalent form of the answer is equally acceptable. Be alert for correct but unfamiliar or unexpected methods — often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. (This does not however apply if candidates have been directed to answer a question by a particular method.) Extra attention to the working is also advisable when the final result is given in the question, indicated by the abbreviation AG in the mark scheme.
5. Where a candidate has crossed out a complete part of a question, it should be marked provided that it has not been replaced. If two or more methods are offered, mark the most complete method. Most complete means the method that has progressed the furthest and is nearest to obtaining a final answer. If two or more complete methods are offered, mark the 'final' method. The final method is either towards the bottom of the answer space or to the right-hand side of the answer space. If a candidate's answer to an item is in two parts, it is important that it is marked as a whole and appropriately annotated.
6. Award NR (No Response)
 - if there is nothing written at all for a question (part), or
 - if there is only a comment which does not in any way relate to the question being asked (e.g. 'can't do', 'don't know')
 - if there is only a mark which is not an attempt at the question (e.g. a dash, a question mark).

Award 0 marks




- if there is an attempt that earns no credit, including copying out the question.

7. Occasionally a candidate will genuinely misread a number in a question and use that value consistently throughout. Indicate such a misread by the MR annotation. Provided that the object and demand of the question remains unaltered, M marks may still be awarded and where A and B marks are available one A mark or one B mark is not awarded. This is only applied once in a whole question. MR is not applied when the candidate misreads his or her own figures – this is regarded as an error in accuracy and marked accordingly.
Example scenario 1: If a candidate misreads a value in part (a) and then continues to use the same misread value in part (b), then the misread rule is only applied to part (a) and all marks in part (b) are available (followed through).
Example scenario 2: If a candidate misreads a value and uses it in only one part of a question e.g. in part (a) the value is misread but not in part (b) or vice versa, then the misread rules are applied to part (a) and part (b) is marked as normal in line with the mark scheme.
8. When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
9. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifies otherwise.
10. Full correct working is required in order to gain accuracy marks. Candidates are required to present the mathematical steps leading to a solution using mathematical notation and not calculator commands. **EXCEPTION:** For SEAB Higher syllabuses, unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
11. Candidates are required to give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. **For international syllabuses**, credit is withheld only once in a paper for answers which are not stated to the minimum accuracy required (minimum accuracy being where an answer is given to 2 significant figures, rather than 3 significant figures).
12. For a non-exact numerical answer whose required accuracy is not specified in the mark scheme, allow the A or B mark if a value is obtained which is correct to 3 significant figures or which would be correct to 3 significant figures if rounded (1 decimal place in the case of an angle). An A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10ms^{-2} .

Report

Please send a brief report on the work of your candidates to the Principal Examiner **by the date specified by the Principal Examiner**, copied to your Team Leader. Your report should cover candidates' responses to each question (or the set of questions given to you by the Principal Examiner) and any other matters that you wish to draw to the attention of Cambridge International or Centres. Consult previous years' reports for the type of comments that are appropriate. These are available on the Cambridge International website for international syllabuses.

Annotations In RMA3

Abbreviation	Annotation in RMA3	Meaning
		Correct work.
		Incorrect work.
M	M0 M1	Method mark.
A	A0 A1	Accuracy mark, awarded for a correct final answer or intermediate stage. They depend on the preceding M mark(s), hence M0 A1 is not possible.
B	B0 B1	Mark awarded for a correct result or statement independent of method marks.
SC	SC	Special case or specific wrong solution for which marks may be awarded, only when indicated in mark scheme.
CWO or NFWW	XP	Correct Working Only or Not From Wrong Working, especially where ‘fortuitous’ answers are seen.
FT	FT	Work can be followed through after an error.
ISW	ISW	Ignore subsequent working (after correct answer obtained).
MR	MR	Misread.
BOD	BOD	Benefit of the doubt is given to the candidate.
		Omission sign.
Pre	Pre	Premature approximation – this applies to A or B marks only.
SF	SF	Error in number of significant figures.
Seen	SEEN	To show that blank pages have been checked for any candidate work.

BP	BP	To show that blank pages have been checked for any candidate work.
O		See other solution

Question	Answer	Marks	Guidance														
1(a)	<table><tr><td>Upper value</td><td>60</td><td>90</td><td>110</td><td>140</td><td>180</td><td>240</td></tr><tr><td>cf</td><td>4</td><td>12</td><td>26</td><td>51</td><td>58</td><td>60</td></tr></table>	Upper value	60	90	110	140	180	240	cf	4	12	26	51	58	60	B1	Cumulative frequencies calculated – may be plotted accurately on graph
	Upper value	60	90	110	140	180	240										
	cf	4	12	26	51	58	60										
	Indicative cf graph inserted	M1	At least 3 points plotted accurately at class upper end points. Linear cf scale $0 \leq cf \leq 60$, linear rime scale $30 \leq \text{time} \leq 240$ with at least 3 values identified on each axis.														
		A1	All points plotted correctly, curve drawn (within tolerance) and joined to (30, 0), axes labelled ‘cumulative frequency’ and ‘hrs [of sunshine]’ o.e.														
3																	
1(b)	$[60 \times 0.7 =] 42$	M1	May be implied by use on graph														
	126	A1FT	Anticipate $123 < \text{hours} < 129$ (tbc), indication of use of graph required. FT from increasing cf graph only read at 42 only.														
		2															
1(c)	Midpoints: 45, 75, 100, 125, 160, 210	B1	At least 5 correct mid-points seen, may be by data table or used in formula.														

	[Mean =] $\frac{4 \times 45 + 8 \times 75 + 14 \times 100 + 25 \times 125 + 7 \times 160 + 2 \times 210}{60} \left[= \frac{6845}{60} \right]$						M1	Correct formula for mean using midpoints ± 0.5 implied by $\frac{6845}{60}$ if 6 correct midpoints seen.												
	= 114						A1	Accept $114\frac{1}{12}$, or 114.083 to at least 3sf.												
							3													
2(a)	$0.6(0.5)^3 + 0.4(0.5)^3 \times 3$						B1	Either $0.6(0.5)^3 + a$ or $b + 0.4(0.5)^3 \times 3, a, b > 0$												
							M1	$0.6(0.5)^3 + 0.4(0.5)^3 \times d$ seen, $d = 1, 2, 3$												
	= 0.225						A1	AG full supporting working required												
							3													
2(b)	<table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X = x)</td><td>0.05</td><td>0.225</td><td>0.375</td><td>0.275</td><td>0.075</td></tr></table>						x	0	1	2	3	4	P(X = x)	0.05	0.225	0.375	0.275	0.075	*B1	Either $P(2) = 0.375, \frac{3}{8}$ or $P(3) = 0.275, \frac{11}{40}$ correct Condone not in table
	x	0	1	2	3	4														
P(X = x)	0.05	0.225	0.375	0.275	0.075															
							B1 FT	Both values in table. FT $P(2) + P(3) = 0.650$												

		2	
2(c)	$\text{Var}(X) = 0.225 + 2^2 \times \text{their } 0.375 + 3^2 \times \text{their } 0.275 + 4^2 \times 0.075 - 2.1^2$ $[= 0.225 + 1.5 + 2.475 + 1.2 - 4.41]$	M1	Correct formula for variance method from their probability distribution table with at least 4 terms, $0 \leq \text{their } P(x) \leq 1$. Accept partially evaluated from correct.

	$[5.4 - 2.1^2] = 0.99[0]$	A1	If M0 awarded SC B1 for 0.99[0] www
		2	
3(a)	Method 1		
	$[P(X > 17) = P(18, 19, 20) =]$ ${}^{20}C_{18} (0.8)^{18} (0.2)^2 + {}^{20}C_{19} (0.8)^{19} (0.2)^1 + {}^{20}C_{20} (0.8)^{20}$	M1	One term ${}^{20}C_x (p)^x (1 - p)^{20-x}$, $0 < p < 1$, $x \neq 0, 20$
	0.13691 + 0.05765 + 0.01153	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.206	B1	$0.206 \leq p \leq 0.2061$
		3	
	Method 2		
	$[P(X > 17) = 1 - P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17) =]$ ${}^{20}C_0 (0.8)^0 (0.2)^{20} + {}^{20}C_1 (0.8)^1 (0.2)^{19} + {}^{20}C_2 (0.8)^2 (0.2)^{18} + \dots + {}^{20}C_{16}$ $(0.8)^{16} (0.2)^4 + {}^{20}C_{17} (0.8)^{17} (0.2)^3$	M1	One term ${}^{20}C_x (p)^x (1 - p)^{20-x}$, $0 < p < 1$, $x \neq 0, 20$
	$1 - (1.048 \times 10^{-14} + 8.389 \times 10^{-13} + 3.188 \times 10^{-11} + \dots + 0.2182 + 0.2054)$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer. If answer correct, condone omission of up to 15 middle terms.
	0.206	B1	$0.206 \leq p \leq 0.2061$
		3	
3(b)	$[(0.8)^4 (0.2) =] 0.0819$	B1	
		1	
3(c)	$(0.8)^5 (0.2)^2 \times 6$	M1	$(0.8)^5 (0.2)^2 \times k$ or $(0.8)^5 (0.2) \times k \times 0.2$

			k an integer 2 – 6
	$= 0.0786$	A1	0.0786432 correct to at least 3SF If M0 awarded, SCB1 for correct answer www
		2	
4	$(1 - x) \times 0.7 \times 0.9 = 0.36$	M1	$(1 - x) \times a \times b = 0.36$, $a = 0.7$ or 0.3 , $b = 0.9$ or 0
		A1	Correct equation formed
	$x = \frac{3}{7}$	A1	Accept 0.428571 to at least 3SF
		3	

Question	Answer	Marks	Guidance
5	$P(A) = \frac{1}{2}$, $P(B) = \frac{8}{24} = \frac{1}{3}$,	B1	Both stated, accept unsimplified.
	$P(A \cap B) = \frac{1}{6}$	M1	Justified
	$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ so events are independent	A1	Evaluated and conclusion stated.
		3	
6(a)	$[P(X < 74) =] P\left(Z < \frac{74-62.3}{8.4}\right) [= P(Z < 1.393)]$	M1	Use of \pm standardisation formula with 74, 62.3 and 8.4 substituted appropriately, not σ^2 , not $\sqrt{\sigma}$, no continuity correction
	$= 0.918$	A1	$0.918 \leq p < 0.9815$ If M0 awarded, SCB1 for correct answer www
		2	

Question	Answer	Marks	Guidance
6(b)	$P(50 < X < 74) = P(-1.464 < Z < 1.393)$	M1	Use of \pm standardisation formula with both 74 (may be seen in 6(a)) & 50, 62.3 and 8.4 substituted appropriately Condone σ^2 , $\sqrt{\sigma}$ and continuity correction ± 0.5
	$[\Phi(1.464) + \Phi(1.393) - 1]$ $0.9285 + 0.9182 - 1$	M1	Calculating the appropriate probability area using z-values (leading to <i>their</i> final answer > 0.5)
	$= 0.847$	A1	$0.8465 < p < 0.8475$. Only dependent on 2 nd M mark, so M0M1A1 possible SCB1 for $0.8465 < p < 0.8475$ if M0A0 awarded
	$(0.8467)^4 = 0.514$	B1 FT	FT <i>their</i> 4-figure probability to power 4
		4	
6(c)	$z_1 = \frac{36-\mu}{\sigma} = -0.739$ $z_2 = \frac{54-\mu}{\sigma} = 1.282$	B1	$-0.740 < z_1 < -0.738$ or $0.738 < z_1 < 0.740$
		B1	$z_1 = \pm 1.282$ (critical value)
		M1	Use of the \pm standardisation formula once with μ , σ and a z-value (not 0.23, 0.77, 0.90, 0.10, ... (add extra)). Condone continuity correct ± 0.5 , not σ^2 , $\sqrt{\sigma}$
	Solve, obtaining values for μ and σ $\mu = 42.6$, $\sigma = 8.91$	M1	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
		A1	
		5	
7(a)	Method 1 Arrangements with 3 Es together – arrangements with 3 Es together and 2 Ds together		

Question	Answer	Marks	Guidance
	$\frac{7!}{2!} - 6!$	B1	$\frac{7!}{2!} - e$, e a positive integer (including 0)
		M1	$f - g$, $f > g$, $g = 6!$ or $7!$
		M1	$\frac{h}{2!} - q$, $\frac{h}{2!} > q$, h & q positive integers
	1800	A1	
	Method 2 identified scenarios ^ EEE ^ ^ ^		
	$5! \times \frac{6 \times 5}{2}$	B1	$5! \times j$, j a positive integer ($j = 1$ may be implied)
		M1	$k! \times \frac{6 \times 5}{n}$ or $k! \times \frac{7 \times 6}{n}$, k a positive integer ($k = 1$ may be implied), $n = 1, 2, 3$
		M1	$k! \times \frac{m \times (m-1)}{2}$ k a positive integer, $m = 10, 9, 8, 7, 6$,
	1800	A1	
		4	
7(b)	[Total number of arrangements =] $\frac{9!}{3!2!} = 30240$	B1	
	[Number of arrangements with 2 Ds in one position with 4 letters in between =] $\frac{7!}{3!} \times 4$	M1	$\frac{7!}{r!} \times s$, $r = 2, 3$ and $s = 1, 2, 3, 4, 5$
		M1	$\frac{t!}{r!} \times 4$, $r = 1, 2, 3$ and $t = 8, 7, 6$
	3360	A1	
	Probability = $\frac{3360}{30240} = \frac{1}{9}$	B1FT	FT $\frac{\text{their } 3360}{\text{their } 30240}$
		5	

Question	Answer	Marks	Guidance
7(c)	Scenarios	B1	Correct outcome/value for one identified scenario, accept unsimplified, www
	D E _ _ _ 4C_3 4	M1	Add values of 5 or 6 appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified, condone use of permutations.
	D D E _ _ 4C_2 6		
	D D E E _ 4C_1 4		
	D E E _ _ 4C_2 6		
	D E E E _ 4C_1 4		
	D D E E E $[{}^4C_0]$ 1		
	[Total =] 25	A1	
		3	

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